Density and spin response function of a normal Fermi gas at unitarity

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Using Landau theory of Fermi liquids we calculate the dynamic response of both a polarized and unpolarized normal Fermi gas at zero temperature in the strongly interacting regime of large scattering length. We show that at small excitation energies the *in phase* (density) response is enhanced with respect to the ideal gas prediction due to the increased compressibility. Viceversa, the *out of phase* (spin) response is quenched as a consequence of the tendency of the system to pair opposite spins. The long wavelength behavior of the static structure factor is explicitly calculated. The results are compared with the predictions in the collisional and superfluid regimes. The emergence of a spin zero sound solution in the unpolarized normal phase is explicitly discussed.

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There is now clear experimental evidence that, in the unitary limit of large scattering lengths, an harmonically trapped polarized Fermi gas phase separates, at very low temperature, into a superfluid core sorrounded by a polarized normal component [1, 2, 3]. When the total polarization of the gas exceeds a critical value the superfluid component disappears and the gas becomes normal. The observed phase separation and the shape of the density profiles in large atomic samples is theoretically well understood employing the equation of state of the superfluid and normal phases in the local density approximation [4, 5]. These studies have revealed the crucial role played by the interactions in the normal phase. The maximum local concentration (Chandrasekhar-Clogston limit) of the minority (spin down) versus the majority (spin up) component achieved at unitarity is predicted to be $x = n_\downarrow/n_\uparrow \sim 0.45$ at zero temperature, in agreement with the most recent experiments carried out at MIT [3]. Larger concentrations are expected to occur if one imposes an adiabatic rotation to the gas which favours the formation of the normal phase [6]. Actually in the rotating case even an unpolarized Fermi gas is expected to give rise, at unitarity, to phase separatation.

The availability of the normal phase of the unitary Fermi gas at very low temperatures opens new stimulating perspectives, due to the expected Fermi liquid behavior of this strongly interacting system. Previous studies of the dynamic behavior have mainly focused on the particle (spectral) response (see, for example, [7] and references therein), for which relevant information is experimentally available through radio-frequency transitions [8]. A quantitative comparison between theory and experiment is however still an open issue, especially due to the non trivial role played by final state interactions. The motion of single impurities in harmonically trapped configurations has also been the object of theoretical investigation [5].

The purpose of this letter is to study the density and spin density dynamic response function of the polarized normal phase. With respect to the motion of single impurities, the excitations investigated in the present work involve wavelengths much smaller than the size of the gas and they consequently correspond to more favourable conditions for reaching the collisionless regime where the most interesting features of the Fermi liquid behavior show up. Experimentally the dynamic response can be measured via two-photon Bragg spectroscopy, a technique already successfully applied to Bose-Einstein condensed gases [9] (first Bragg spectroscopic measurements in ultracold Fermi gases have been recently carried out at high momentum transfer [10]). Differently from *rf* transitions Bragg experiments do not change the internal atomic states and consequently final state effects are absent. Furthermore, by a proper choice of the detuning of the laser beams with respect to the atomic resonance one can measure suitable combinations of the density and spin responses. Finally, using focused laser beams, one can in principle measure the response locally, thereby providing valuable information on the uniform matter behavior. Theoretically the density and spin responses of the normal phase can be calculated using Landau theory of Fermi liquids [11]. This theory has been so far mainly applied to unpolarized samples like liquid He3. It has been also developed to study spin transverse excitations in slightly polarized samples [12, 13]. In this work we consider arbitrary polarized Fermi gases.

The basic assumption of Landau's theory is that the system can be described in terms of a gas of long-living quasi-particles interacting through a mean field. This assumption is usually guaranteed at sufficiently low temperature for excitations close to the Fermi surface of each spin component. Under these assumptions collisions between quasi-particles can be ignored. A key feature of the theory is the occurrence of deformations of the Fermi surface which deeply distinguish the dynamic behavior of a normal liquid from the one of a superfluid. In some cases these deformations give rise to a new type of collective motion, the so-called zero sound. For sake of simplicity in the following we will take into account only the interaction terms arising from the isotropic deformations of the Fermi surface, i.e. from the density modulations of the two spin components. In particular we will ignore effective mass effects which are predicted to be small in the polarized phase at unitarity [5, 14, 15, 16]. According to the above assumption the relevant interaction terms can

be derived from the interaction energy functional

$$E_{int} = \int d\mathbf{r}\; e_{int}(n_\uparrow, n_\downarrow)\ . \quad (1)$$

The single quasi-particle (qp) Hamiltonian for each σ-spin component ($\sigma =\uparrow,\downarrow$) will then contain an interaction contribution easily derivable from Eq.(1): $H_\sigma^{(qp)} = H_0 + \partial e_{int}/\partial n_\sigma$ where H_0 is the free particle Hamiltonian. The quasi-particle Hamiltonian will be modified during the motion by position and time dependent terms, associated with the local changes in the density distribution of the two spin species. These terms have to be determined with a self-consistent procedure. In the linear approximation one has

$$H_\sigma^{(qp)} = H_0 + \left(\frac{\partial e_{int}}{\partial n_\sigma}\right)_0 + \left(\frac{\partial^2 e_{int}}{\partial n_\sigma \partial n_\uparrow}\right)_0 \delta n_\uparrow + \left(\frac{\partial^2 e_{int}}{\partial n_\sigma \partial n_\downarrow}\right)_0 \delta n_\downarrow \quad (2)$$

where the suffix 0 indicates that the derivatives should be calculated at equilibrium and $\delta n_\sigma \equiv \delta n_\sigma(\mathbf{r},t)$ are the space and time dependent density changes with respect to equilibrium. The dynamic response function is calculated by adding an external field of the form $V_{ext} = \alpha_\sigma e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$ to the Hamiltonian (2) and the linear response $\chi_{\sigma,\sigma'}$ is defined by the induced density fluctuations according to

$$\delta n_\sigma(\mathbf{r},t) = \sum_{\sigma'} \alpha_{\sigma'} \chi_{\sigma,\sigma'}(q,\omega) e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\ . \quad (3)$$

By inserting result (3) into Eq.(2) we can calculate the response of the system using the free particle Hamiltonian by simply replacing the external coupling α_σ with the effective value

$$\alpha_\sigma^{eff} = \alpha_\sigma + \sum_{\sigma'} \alpha_{\sigma'} \left(\frac{\partial^2 e_{int}}{\partial n_\sigma \partial n_{\sigma'}}\right)_0 \chi_{\sigma,\sigma'} \quad (4)$$

in V_{ext}. This procedure yields a self-consistency relationship for the dynamic response functions and provides the relevant expressions for $\chi_{\sigma,\sigma'}$ in terms of the free particle responses $\chi^0_{\uparrow,\uparrow}$ and $\chi^0_{\downarrow,\downarrow}$ (the crossed free response $\chi^0_{\uparrow,\downarrow}$ identically vanishes because there are no correlations between spin-up and spin-down particles in the ideal gas):

$$\begin{aligned}
\chi_{\uparrow,\uparrow}(q,\omega) &= \chi^0_{\uparrow,\uparrow}(q,\omega)\left[1 - \chi^0_{\downarrow,\downarrow}(q,\omega)\frac{\partial^2 e_{int}}{\partial n_\downarrow \partial n_\downarrow}\right]/D(q,\omega) \\
\chi_{\downarrow,\downarrow}(q,\omega) &= \chi^0_{\downarrow,\downarrow}(q,\omega)\left[1 - \chi^0_{\uparrow,\uparrow}(q,\omega)\frac{\partial^2 e_{int}}{\partial n_\uparrow \partial n_\uparrow}\right]/D(q,\omega) \\
\chi_{\uparrow,\downarrow}(q,\omega) &= \chi^0_{\uparrow,\uparrow}(q,\omega)\chi^0_{\downarrow,\downarrow}(q,\omega)\frac{\partial^2 e_{int}}{\partial n_\uparrow \partial n_\downarrow}/D(q,\omega) \quad (5)
\end{aligned}$$

where the denominator $D(q,\omega)$ is defined by

$$\begin{aligned}
D(q,\omega) &= \left[1 - \chi^0_{\downarrow,\downarrow}(q,\omega)\frac{\partial^2 e_{int}}{\partial n_\downarrow \partial n_\downarrow}\right]\left[1 - \chi^0_{\uparrow,\uparrow}(q,\omega)\frac{\partial^2 e_{int}}{\partial n_\uparrow \partial n_\uparrow}\right] \\
&- \chi^0_{\uparrow,\uparrow}(q,\omega)\chi^0_{\downarrow,\downarrow}(q,\omega)\left(\frac{\partial^2 e_{int}}{\partial n_\uparrow \partial n_\downarrow}\right)^2 \quad (6)
\end{aligned}$$

and, for simplicity, we have omitted the suffix 0 in the density derivatives of e_{int}. The effects of the interaction is particularly crucial in the crossed $\chi_{\uparrow,\downarrow}$ response. As we will explicitly discuss later the sign of the relevant interaction term $\partial^2 e_{int}/\partial n_\uparrow \partial n_\downarrow$ is negative, reflecting the tendency of the system to pair opposite spins. The poles of the response functions (zeros of $D(q,\omega)$) correspond to the undamped, discretized oscillations of the system (zero sound). They occur only in the presence of the interaction. Useful combinations of (5) are provided by the symmetric (s) and antisymmetric (a) responses $\chi_{s(a)} \equiv \chi_{\uparrow,\uparrow} + \chi_{\downarrow,\downarrow} \pm 2\chi_{\uparrow,\downarrow}$, also called density and spin responses.

The concept of quasi-particle, and consequently result (5) for the response functions, applies to small wave vectors ($q \ll q_{F\sigma}$ where $q_{F\sigma}$ is the Fermi wavevector of the σ-specis). In this limit the free response χ^0, at zero temperature, reduces to the simple form:

$$\chi^0_{\sigma,\sigma'}(q,\omega) = -\frac{m q_{F\sigma}}{2\pi^2} g(\lambda_\sigma)\delta_{\sigma,\sigma'} \quad (7)$$

with $\lambda_\sigma = \omega/q v_{F\sigma}$ and $v_{F\sigma} \equiv \hbar q_{F\sigma}/m$. The function $g(\lambda)$ is defined by [11]

$$\begin{aligned}
g(\lambda) &= 1 - \frac{\lambda}{2}\ln\frac{1+\lambda}{1-\lambda} - i\frac{\lambda}{2}\pi \quad if \quad 0<\lambda<1 \\
g(\lambda) &= 1 - \frac{\lambda}{2}\ln\frac{\lambda+1}{\lambda-1} \quad if \quad \lambda > 1\ . \quad (8)
\end{aligned}$$

From the knowledge of the response function one can calculate the dynamic structure factor in each spin channel. At zero temperature the following relation holds for $\omega > 0$

$$S_\sigma(q,\omega) = -\frac{1}{\pi} Im\chi_{\sigma,\sigma}(q,\omega)\ , \quad (9)$$

while $S_\sigma(q,\omega) = 0$ for $\omega < 0$. The function $S_\sigma(q,\omega)$ consists, in general, of a discretized peak (zero sound) and of a continuum of quasi-particle excitations. In the absence of interactions one finds $S^0_\sigma(q,\omega) = \lambda_\sigma m/2\pi^2$ for $0 < \lambda_\sigma < 1$ and 0 elsewhere.

The dynamic structure factor obeys important sum rules [11, 17]. The most famous one is the f-sum rule $\int d\omega\omega S_\sigma(q,\omega) = n_\sigma q^2/2m$ which is exactly satisfied by the mean field results (5) for $\chi_{\sigma,\sigma}$, being directly related to the $1/\omega^2$ behavior of the response function at large ω. Another important sum rule is provided by the non-energy weighted moment, yielding the static structure factor $\int d\omega S_\sigma(q,\omega) = S_\sigma(q)$. This quantity is directly related to the Fourier tansform of the two-body correlation function. Landau's theory provides accurate information on the long wavelength (small q) behavior where $S_\sigma(q)$ is linear in q.

The simplest case is the symmetric configuration ($n_\uparrow = n_\downarrow$). At very low temperature the corresponding ground state configuration is expected to be superfluid, the normal phase being available only at higher temperatures where Landau's theory is not applicable. However, as already anticipated in the introduction, if one

switches on adiabatically the rotation of the confining trap, also in the unpolarized case the gas is predicted to phase separate at zero temperature into a superfluid core sorrounded by a normal shell [6], so that the study of the dynamic behavior of the unpolarized normal gas might be relevant for future experiments. In this case the interaction coefficients determining the response function are conveniently written as $\partial^2 e_{int}/\partial n_\uparrow \partial n_\uparrow = \partial^2 e_{int}/\partial n_\downarrow \partial n_\downarrow = 2/3(\epsilon_F/n)(F_0^s + F_0^a)$ and $\partial^2 e_{int}/\partial n_\uparrow \partial n_\downarrow = 2/3(\epsilon_F/n)(F_0^s - F_0^a)$ where $F_0^{s(a)}$ are the Landau parameters in terms of which the density (s) and spin (a) response functions take the familiar form [11]

$$\chi_{s(a)} = -\frac{mq_F}{\pi^2}\frac{g(\lambda)}{1+F_0^{s(a)}g(\lambda)}\,. \tag{10}$$

The above Landau's parameters are directly related to the compressibility $1/mc^2$ and to the magnetic susceptibility χ_M of the gas according to

$$\begin{aligned} mc^2 &= mc_0^2(1+F_0^s) \\ \chi_M &= \chi_{M0}/(1+F_0^a) \end{aligned} \tag{11}$$

where $c_0 = v_F/\sqrt{3}$ and χ_{M0} are, respectively, the sound velocity and the magnetic susceptibility of the ideal gas. Since the gas is strongly interacting the evaluation of F_0^s and F_0^a requires a non perturbative approach. Monte Carlo calculations of the equation of state and of the magnetic susceptibility in the normal phase yield the values $F_0^s = -0.44$ [18] and $F_0^a \sim 2$ [19] at unitarity [20]. The behavior of the Landau's parameters of the unitary Fermi gas deeply differs from the one of liquid ^3He where F_0^a is negative reflecting the tendency of the system to behave like a ferromagnet. In our case F_0^a is instead positive, reflecting the tendency of the system to pair opposite spins. An interesting consequence is the appearence of a discretized pole in the spin response, associated with the propagation of spin zero sound, while the density response is Landau damped. In Fig.1 we show the predicted values of the density and spin dynamic structure factor in the unpolarized case, using the values $F_0^s = -0.45$ and $F_0^a = 2.0$. The spin zero sound solution takes place at the value $\omega = 1.16 q v_F$. The different behavior in the density and spin channels is evident and shows up also in the low q behavior of the static structure factors: $S_s(q) = 1.26 S_0(q)$ and $S_a(q) = 0.62 S_0(q)$ where $S_0(q) = 3/4 nq/q_F$ is the ideal Fermi gas prediction and n is the gas density. The strength carried by the zero sound is ~ 80 percent of the total spin strength.

In the polarized case a first useful description of the interaction effects is provided by the low x expansion of the equation of state. At unitarity the expansion takes the simple form $e_{int} = -(3/5)A\epsilon_{F\uparrow}n_\downarrow$ with the value $A \sim 1$ predicted by both Monte Carlo [5, 15, 16] and diagrammatic calculations [14] and where $\epsilon_{F\uparrow}$ is the Fermi energy of the spin-up component. The expansion has been shown to provide an accurate description of the equation of state up to the largest values of concentration achievable in the absence of rotation. In the small x regime the relevant interaction parameter is $\partial^2 e_{int}/\partial n_\uparrow \partial n_\downarrow = -2/5 A\epsilon_{F\uparrow}/n_\uparrow$.

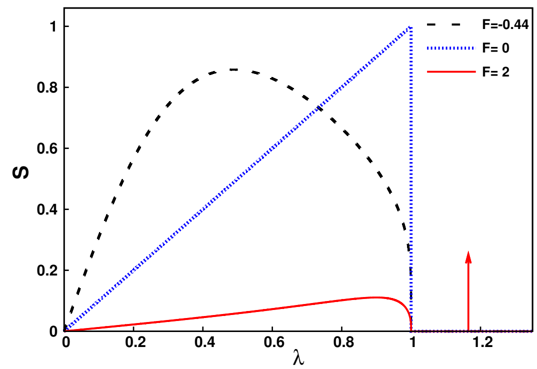

FIG. 1: (Color online) Density (dashed, black) and spin (full line, red) dynamic structure factor for an unpolarized Fermi gas in the normal phase at unitarity and $T = 0$ as a function of $\lambda = \omega/qv_F$. The red arrow indicates the position of the spin zero sound solution. The ideal gas prediction is also shown (dotted, blue).

In Fig.2 we show the predictions for the density and spin dynamic structure factors calculated for a polarized Fermi gas at $x = 0.44$. The comparison with the ideal gas prediction (dotted line) explicitly reveals the role of the interactions. Similarly to the unpolarized case interactions have opposite effects in the two channels. For the static structure factors, proportional to the integral of the dynamic structure factor, we find the results $S_s(q) = 1.28 S_0(q)$ and $S_a(q) = 0.83 S_0(q)$ where $S_0(q) = 3/4(1 + x^{2/3}) n_\uparrow q/q_{F\uparrow} = 1.18 n_\uparrow q/q_{F\uparrow}$ is the ideal gas value. The cusps in the dynamic structure factors at $\lambda_\uparrow = 1$ and $\lambda_\uparrow = x^{1/3}$ reflect the existence of two Fermi surfaces in the polarized case. In addition to the continuous structure the response in the spin channel gives also rise to a discretized contribution (spin zero sound). This pole is however located too close to the continuum threshold $\lambda_\uparrow = 1$ to be physically relevant, differently from what happens in the unpolarized case.

The above results for the response function hold in the collisionless regime of the normal phase. When collisions become important the dynamic behavior changes in a drastic way, being characterized by an ordinary (first) sound mode, giving rise to a sharp peak in the density response, and by a diffusive spin excitation. The first sound velocity is determined by the compressibility of the gas according to the thermodynamic relation $mc^2 = n\partial\mu/\partial n$. For the unpolarized case we find $c = v_F(1 + F_0^s)^{1/2}/\sqrt{3}$, corresponding to $\lambda = \omega/qv_F = 0.43$, while for the polarized gas one finds $c = v_{F\uparrow}\left((1 + x^{5/3} - Ax)/(1 + x)\right)^{1/2}/\sqrt{3}$ [21]. At $x = 0.44$ the reduction of the sound velocity with respect to the ideal gas ($A = 0$) is still significant and comparable to the reduction calculated in the unpolarized case. It is also useful to compare the above pre-

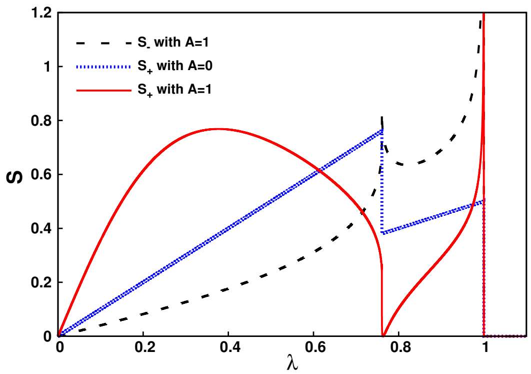

FIG. 2: (Color online) Density (dashed, black) and spin (full line, red) dynamic structure factor for a normal polarized ($x = n_\uparrow/n_\downarrow = 0.44$) Fermi gas at unitarity and $T = 0$ as a function of $\lambda = \omega/qv_{F\uparrow}$. The ideal gas prediction is also shown (dotted, blue).

dictions with the superfluid behavior of the upolarized gas where sound propagates hydrodynamically at the velocity $c = v_F(1+\beta)^{1/2}/\sqrt{3}$ with $\beta \sim= -0.58$. In the superfluid the spin excitations are instead gapped with the threshold given by 2Δ where Δ is the single particle gap, of the order of the Fermi energy at unitarity.

The transition from the collisionless to the collisional regime takes place when $\omega\tau \sim 1$ where τ is a typical collisional time. Using dimensional arguments one expects, at unitarity, the dependence $\hbar/\tau \propto \epsilon_F(k_BT/\epsilon_F)^2$, where the T^2 factor originates from the Pauli principle. The value of the coefficient of proportionality is estimated to be ~ 4 both in the unpolarized case [22] and in the highly polarized limit [23] (in the latter case ϵ_F is the Fermi energy of the majority component). The conditions for applying Landau's theory in the collisionless regime are not consequently too severe, requiring frequencies satisfying the condition:

$$\epsilon_F \left(\frac{k_BT}{\epsilon_F}\right)^2 \ll \hbar\omega \ll \epsilon_F \, . \tag{12}$$

The condition $\hbar\omega \gg \epsilon_F\,(k_BT/\epsilon_F)^2$ is in particular much less severe than in the case of the spin dipole oscillation of the trapped gas which takes place at frequencies of the order of the harmonic oscillator frequency [5].

In conclusion we have shown that at unitarity the response function of a Fermi gas in its normal phase is sizably affected by the interactions and behaves quite differently in the density and spin channels. Both mean field and collisional effects are predicted to take place in ranges of temperatures and frequencies of reasonably easy access in two photon Bragg spectroscopy experiments.

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[21] It is worth noticing that in the polarized case the compressibility does not coincide with the static limit of the density response function. The calculation of the compressibility actually assumes the adiabatic condition $n_\uparrow/n_\downarrow = const$ in the calculation of the density derivative of the chemical potential.

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